

Wave-Packet Analysis of Single-Slit Ghost Diffraction

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Abstract. We show that single-slit two-photon ghost diffraction can be explained very simply by using a wave-packet evolution of a generalised EPR state. Diffraction of a wave travelling in the x-direction can be described in terms of the spreading in time of the transverse (z-direction) wave-packet, within the Fresnel approximation. The slit is assumed to truncate the transverse part of the wavefunction of the photon to within the width of the slit. The analysis reproduces all features of the two-photon single-slit ghost diffraction.

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1 Introduction

The issue of quantum nonlocality which results from entangled states, has been a subject of debate since the time it was brought to prominence by the seminal paper of Einstein, Podolsky and Rosen (EPR)[1]. Quantum systems which are said to be entangled, show certain correlation in their measurement results even though they may be far separated in space,[2]. This is a feature of quantum mechanics which many find discomfoting.

A dramatic demonstration of quantum nonlocality is in the so-called ghost interference experiment by Strekalov et.al.[3]. We describe below, the single-slit ghost diffraction experiment. An entangled pair of photons emerges from a spontaneous parametric down-conversion (SPDC) source S. Photons, which we shall refer to as 1 and 2, are separated by a beam-splitter and travel in different directions. A single-slit is kept in the path of photon 1, and there is a movable detector D1 behind it (see Fig. 1). Normally one would expect a single-slit diffraction pattern. No first order diffraction is observed for photon 1. This result is unexpected because one would expect photons from a laser, passing through a single-slit, to undergo diffraction. The second, and the more surprising result of the experiment is that when photon 2 is detected by detector D2, *in coincidence* with a *fixed* detector D1 detecting photon 1, photon 2 shows a single-slit diffraction pattern. Photon 2 does not pass through any slit, and one would not expect it to show any diffraction. Another curious feature of this ghost-diffraction is that the fringe width of the diffraction pattern follows a diffraction formula $w = \frac{\lambda D}{\epsilon}$, where λ is the wavelength of the photons, ϵ the width of the slit, and D is a very curious distance

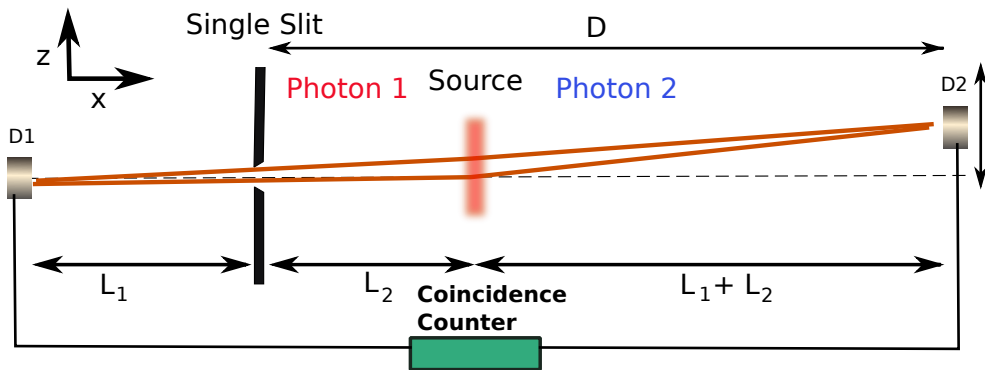


Fig. 1. Schematic diagram of the single-slit ghost interference experiment. Entangled photons 1 and 2 emerge from the SPDC source S and travel in different directions along x-axis.

from slit, right through the SPDC source S to the detector D2. Note that photon 2 does not even travel that much of distance. The experiment attracted a lot of debate and research attention [4, 5, 6, 7, 8, 9, 10, 11, 12].

The double-slit version of this experiment has been understood as a combined effect of a virtual double-slit creation for photon 2, and *quantum erasure* of which-path information[10, 11, 12]. In that analysis the amplitudes of photon 1, to pass through the two slits, are correlated with certain amplitudes of photon 2. However, the correlation between the two photons within the narrow region of one slit had been ignored. In order to explain single-slit ghost diffraction, one would need to take into account the entanglement between the photons within the region of a single slit. A rigorous theoretical analysis of the single-slit ghost experiment would involve introducing a two-photon entangled state, and evolving it in time in the presence of single-slit potential. This is a challenging task.

It has been shown earlier that diffraction of light can be described in terms of evolution of de Broglie wave-packets[13, 14]. Single-slit diffraction of light in particular, has been described in terms of Gaussian wave-packet evolving via Schrödinger evolution[15]. In the following we generalise these methods to the case of an entangled state. We show that the single-slit ghost diffraction can be explained rather simply using this approach.

2 Wave-packet analysis

2.1 The entangled state

We start with a generalised EPR state[16]

$$\Psi(z_1, z_2) = C' \int_{-\infty}^{\infty} dk e^{-k^2/4\sigma^2} e^{-ikz_2} e^{ikz_1} e^{-\frac{(z_1+z_2)^2}{4\Omega^2}}, \quad (1)$$

where C' is a normalization constant, and σ, Ω are certain parameters to be explained later. The state (1), unlike the original EPR state[1], is well behaved and fully normalised. In the limit $\sigma, \Omega \rightarrow \infty$ the state (1) reduces to the original EPR state.

The photons of the pair are assumed to be travelling in opposite directions along the x-axis, but the entanglement is in the z-direction. In our analysis we will ignore the dynamics along the x-axis as it does not affect the entanglement. We just assume that during evolution for a time t , the photon travels a distance equal to ct in the x-direction. Integration over k can be performed in (1) to obtain:

$$\Psi(z_1, z_2) = \sqrt{\frac{2\sigma}{\pi\Omega}} e^{-(z_1-z_2)^2\sigma^2} e^{-(z_1+z_2)^2/4\Omega^2}. \quad (2)$$

The uncertainty in position and the wave-vector of the two photons, along the z-axis, is given by

$$\Delta z_1 = \Delta z_2 = \sqrt{\Omega^2 + 1/4\sigma^2}, \quad \Delta k_{1z} = \Delta k_{2z} = \frac{1}{2} \sqrt{\sigma^2 + \frac{1}{4\Omega^2}}. \quad (3)$$

From the above it is clear that Ω and σ quantify the position and momentum (wave-vector) spread of the photons in the z-direction. Notice however, that for the special case $2\Omega = 1/\sigma$, the state (2) is factored into a product of two Gaussians for z_1 and z_2 alone. Thus the state is not entangled for this particular choice of parameters.

2.2 Time evolution

We first lay out our strategy for the time evolution of a photon wave-packet. Consider the state of a single photon at time $t = 0$ as $\psi(z, 0)$. The state at a later time t is given by

$$\psi(z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp(ik_z z - i\omega(k_z)t) \tilde{\psi}(k_z, 0) dk_z, \quad (4)$$

where $\tilde{\psi}(k_z, 0)$ is the Fourier transform of $\psi(z, 0)$ with respect to z . Now photon is approximately travelling in the x-direction, but can slightly deviate in the z-direction (which allows it to pass through slits located at different z-positions), so that its true wave-vector will have a small component in the z-direction too. So

$$\omega(k_z) = c\sqrt{k_x^2 + k_z^2} \quad (5)$$

Since the photon is travelling along x-axis by and large, we can write $k_x \approx k_0$, where k_0 is the wavenumber of the photon associated with its wavelength, $k_0 = 2\pi/\lambda$. The dispersion along z-axis can then be approximated by

$$\omega(k_z) \approx ck_0 + ck_z^2/2k_0. \quad (6)$$

This is essentially the Fresnel approximation. Using this approximation, eqn. (4) assumes the form

$$\psi(z, t) = \frac{e^{-ick_0 t}}{2\pi} \int_{-\infty}^{\infty} \exp(ik_z z - ickt_z^2/2k_0) \tilde{\psi}(k_z, 0) dk_z \quad (7)$$

Coming back to our problem of entangled photons, we assume that after travelling for a time t_0 , photon 1 reaches the slit ($ct_0 = L_2$), and photon 2 travels a distance L_2 towards detector D2. Using the strategy outlined in the preceding discussion, we can write the state of the entangled photons after a time t_0 as follows:

$$\psi(z_1, z_2, t_0) = \frac{e^{-2ick_0 t_0}}{4\pi^2} \int_{-\infty}^{\infty} dk_1 \exp(ik_1 z_1 - ickt_0 k_1^2/2k_0) \int_{-\infty}^{\infty} dk_2 \exp(ik_2 z_2 - ickt_0 k_2^2/2k_0) \tilde{\psi}(k_1, k_2, 0), \quad (8)$$

where $\tilde{\psi}(k_1, k_2, 0)$ is the Fourier transform of (2) with respect to z_1, z_2 . After some algebra, the above can be worked out to be

$$\psi(z_1, z_2, t_0) = C \exp \left[\frac{-(z_1 + z_2)^2}{4\Omega^2 + 2i\lambda ct_0/\pi} \right] \exp \left[\frac{-(z_1 - z_2)^2}{\frac{1}{\sigma^2} + 2i\lambda ct_0/\pi} \right], \quad (9)$$

where $C = \sqrt{\frac{2}{\pi}} \left[\left\{ \Omega^2 + \left(\frac{\lambda ct_0}{2\pi\Omega} \right)^2 \right\} \left\{ \frac{1}{\sigma^2} + \left(\frac{2\sigma\lambda ct_0}{\pi} \right)^2 \right\} \right]^{-1/4}$. The above equation represents the state of the entangled photons just before photon 1 enters the slit.

2.3 Effect of slit

In order to incorporate the effect of the slit of width ϵ on the entangled state, we just assume that the slit abruptly truncates the state of particle 1 such that only the part $-\epsilon/2 \leq z_1 \leq \epsilon/2$ survives. Consequently we assume this truncated state to be our starting point after emerging from the slit. After emerging from the slit, photon 1 travels for a time t , a distance L_1 , to reach the detector D1, and photon 2 travels for the same time to reach detector D2. Using (7), the time propagation kernel for the two photons can be written as

$$\begin{aligned} K_1(z_1, z'_1, t) &= \sqrt{\frac{1}{i\lambda ct}} \exp \left[\frac{-\pi(z_1 - z'_1)^2}{i\lambda ct} \right] \\ K_2(z_2, z'_2, t) &= \sqrt{\frac{1}{i\lambda ct}} \exp \left[\frac{-\pi(z_2 - z'_2)^2}{i\lambda ct} \right] \end{aligned} \quad (10)$$

The two-particle state after a time t (after t_0) is given by

$$\psi(z_1, z_2, t) = \int_{-\epsilon/2}^{\epsilon/2} dz'_1 K_1(z_1, z'_1, t) \int_{-\infty}^{\infty} dz'_2 K_2(z_2, z'_2, t) \psi(z'_1, z'_2, t_0) \quad (11)$$

Notice that because of the truncation, the wavefunction above is no longer normalised. Subsequently we will continue to treat the unnormalised wavefunction. Integration over z'_2 can be performed straightaway to give

$$\psi(z_1, z_2, t) = C_r \int_{-\epsilon/2}^{\epsilon/2} \exp \left[\frac{-\pi(z_1 - z'_1)^2}{i\lambda ct} \right] \exp \left[\frac{-(z_2 - z'_1(\frac{\Gamma+\gamma}{\Gamma+\gamma}))^2}{\frac{\Gamma\gamma}{\Gamma+\gamma} + \frac{i\lambda ct}{\pi}} \right] \exp \left[\frac{-4z_1'^2}{\Gamma+\gamma} \right] dz'_1 \quad (12)$$

where

$$\Gamma = 4\Omega^2 + \frac{i2\lambda ct_0}{\pi}, \quad \gamma = \frac{1}{\sigma^2} + \frac{i2\lambda ct_0}{\pi}, \quad (13)$$

and $C_r = \frac{C}{i\lambda ct} \left(\frac{\Gamma+\gamma}{\pi\Gamma\gamma} + \frac{1}{i\lambda ct} \right)^{-1/2}$.

Assuming the slit is very narrow, we notice that in the integral above, $|z'_1| \leq \epsilon/2 \ll 1$. Hence we can make the following approximation

$$(z_n - z'_1)^2 \approx z_n^2 - 2z_n z'_1, \quad (14)$$

where z_n is z_1 or z_2 . This amounts to neglecting ϵ^2 in comparison to ϵ , and results in

$$\psi(z_1, z_2, t) \approx C_r \int_{-\epsilon/2}^{\epsilon/2} \exp \left[\frac{i\pi z_1^2 - 2i\pi z_1 z_1'}{\lambda ct} \right] \exp \left[\frac{-z_2^2 + 2z_2 z_1' (\frac{\Gamma-\gamma}{\Gamma+\gamma})}{\frac{\Gamma\gamma}{\Gamma+\gamma} + \frac{i\lambda ct}{\pi}} \right] dz_1'. \quad (15)$$

In (12), the argument of the exponent in the last term is of the order of ϵ^2 , and thus that term has been neglected in writing the above approximate form.

Integration over z_1' can now be carried out with ease, and one obtains

$$\psi(z_1, z_2, t) = C_r \exp \left[\frac{i\pi z_1^2}{\lambda ct} - \frac{z_2^2}{\alpha} \right] \frac{\sin(\epsilon\pi z_1/2\lambda ct - i\epsilon z_2\beta)}{\pi z_1/2\lambda ct - i z_2\beta}, \quad (16)$$

where $\alpha = \frac{\Gamma\gamma}{\Gamma+\gamma} + \frac{i\lambda ct}{\pi}$ and $\beta = \frac{1}{\alpha} \left(\frac{\Gamma-\gamma}{\Gamma+\gamma} \right)$.

The probability density of joint detection of photon 2 at z_2 , and photon 1 at z_1 , is given by

$$|\psi(z_1, z_2, t)|^2 = |C_r|^2 \left| e^{-z_2^2/\alpha} \right|^2 \left| \frac{\sin(\epsilon\pi z_1/2\lambda ct - i\epsilon z_2\beta)}{\pi z_1/2\lambda ct - i z_2\beta} \right|^2. \quad (17)$$

Before we come to the results, we recall that the final expression is understood better in terms of the distance travelled by the photons, rather than the time for which they evolve. In the following analysis, we use $ct_0 = L_2$, $ct = L_1$ and $2L_2 + L_1 = D$.

3 Results

3.1 Ghost diffraction

Ghost diffraction arises when photon 2 is detected at D2 in coincidence with photon 1 being detected at D1 *fixed* at $z_1 = 0$. The probability density of this happening can be evaluated by putting $z_1 = 0$ in (17). Entanglement between the two photons is good when $\Omega, \sigma \gg 1$. In this limit we safely assume $\Omega \gg 1/\sigma$. In the approximation $2\lambda L_2/\pi \gg 1/\sigma^2$, this (unnormalised) probability density assumes the form

$$|\psi(0, z_2, t)|^2 = A \frac{1}{z_2^2} \left[\sin^2 \left(\frac{\pi\epsilon z_2}{\lambda D} \right) \cosh^2 \left(\epsilon z_2 \left(\frac{\pi}{\sigma\lambda D} \right)^2 \right) + \cos^2 \left(\frac{\pi\epsilon z_2}{\lambda D} \right) \sinh^2 \left(\epsilon z_2 \left(\frac{\pi}{\sigma\lambda D} \right)^2 \right) \right], \quad (18)$$

where

$$A = \frac{1}{\sqrt{\Omega^2 + \left(\frac{\lambda L_2}{2\pi\Omega} \right)^2}} \left(\frac{D}{2\pi^2\sigma L_1} \right) e^{-2\left(\frac{\pi z_2}{\sigma\lambda D} \right)^2} \quad (19)$$

The probability density, given by (18), represents a diffraction pattern for photon 2. In order to compare the theoretical results obtained here with the experimental results of Strekalov et.al., (18) has been plotted after normalising and scaling with 500 counts, and using the other parameter values of the experiment (see Fig. 2). The parameter σ has been assigned an arbitrary value 5 mm^{-1} . The name ghost diffraction is natural for this phenomenon, as photon 2 does not pass through any real slit, but still shows diffraction. The distance between the central maximum and the first maximum is given by

$$w = \frac{\lambda D}{\epsilon}. \quad (20)$$

The surprise here is that, taken at face value, the above formula would represent diffraction of photons of wavelength λ emerging from a slit of width ϵ and travelling a distance $D = 2L_2 + L_1$ to reach detector D2. However, photons 2 really travel only a distance $L_1 + L_2$. This is exactly what was seen in the experiment [3]. This quaint feature is the result of entanglement between the two photons.

As a special case, if $2\Omega = 1/\sigma$ we find that $\Gamma = \gamma$ and consequently $\beta = 0$. In the limit $\beta \rightarrow 0$, the probability density of detecting photon 2 at a position z_2 , *in coincidence with D1 fixed at $z_1 = 0$* , is given by

$$|\psi(0, z_2, t)|^2 = |C_r|^2 \left| e^{-z_2^2/\alpha} \right|^2 \epsilon^2. \quad (21)$$

This is just a Gaussian in z_2 and implies that there is no ghost diffraction, which should not be surprising because for $\beta = 0$ the two photons are disentangled.

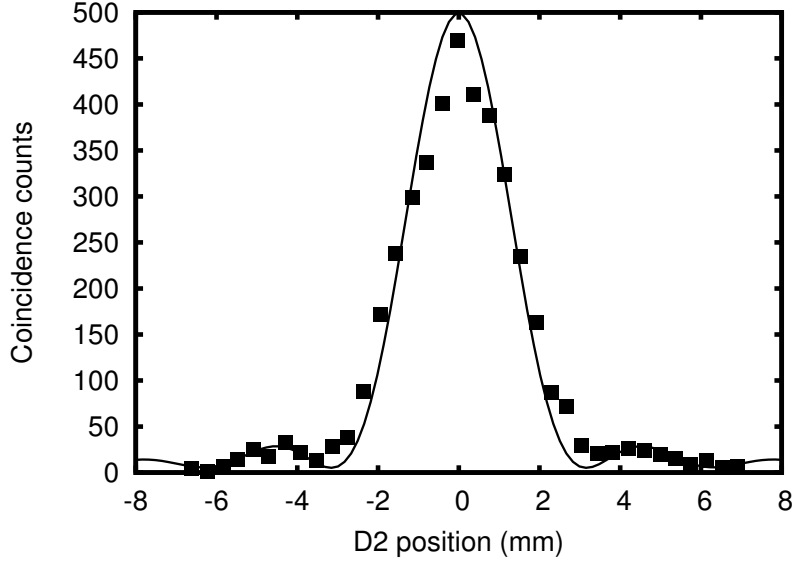


Fig. 2. Normalised probability density of photon 2 (scaled by a factor of 500), conditioned on photon 1 reaching $z_1 = 0$, as a function of the position of detector D2, with D1 fixed at $z_1 = 0$, for $\lambda = 702.2$ nm, $D = 1.8$ m, $\epsilon = 0.4$ mm, and $\sigma = 5.0$ mm⁻¹. Squares represent the experimental data from Strekalov et. al. [3].

Let us try to understand why a coincident count of D2 with a *fixed* D1 is needed for ghost interference to appear. Not doing a coincident count would amount to integrating over z_1 , the position of D2, in (17). It can be easily seen that, for $\beta \neq 0$, integration over z_1 would kill the oscillations in (17), which means no ghost diffraction.

Another way to understand why coincident counting is needed for ghost diffraction is that if D1 were fixed at $z_1 = z_0$ instead of $z_1 = 0$, the term $\left| \frac{\sin(i\epsilon z_2 \beta - \pi \epsilon z_0 / 2\lambda ct)}{i z_2 \beta - \pi z_0 / 2\lambda ct} \right|^2$ would lead to a slightly shifted ghost diffraction pattern. This feature has also been experimentally observed [3]. Corresponding to different values of z_0 , the diffraction pattern would be shifted by different amounts, and a sum of all such possibilities would lead to the washing out of the pattern.

3.2 Missing first order diffraction

Let us now turn our attention to the issue of missing first order single-slit diffraction for photon 1 behind the slit. In the first part of the experiment, photons 1 pass through the single slit and are detected by a scanning detector D1. Normally one would expect single slit diffraction. The authors of the experiment explain it by saying that the blurring out of the first order interference fringes is due to the considerably large angular propagation uncertainty of a single SPDC photon. However, we will show that the real reason lies in the entanglement of photon 1 with photon 2.

The probability of joint detection of the two photons at the two detectors is given by (17). If one wants to look for first order diffraction of photon 1, one has to ignore photon 2 or, in other words, integrate over all values of z_2 . Clearly, if one were to evaluate $\int_{-\infty}^{\infty} |\psi(z_1, z_2, t)|^2 dz_2$ using (17), the oscillations will be gone and it will not yield a diffraction pattern. Interestingly however, for the case $2\Omega = 1/\sigma$ ($\beta = 0$), (17) reduces to

$$|\psi(z_1, z_2, t)|^2 = |C_r|^2 \left| e^{-z_2^2/\alpha} \right|^2 \left| \frac{\sin(\epsilon \pi z_1 / 2\lambda ct)}{\pi z_1 / 2\lambda ct} \right|^2. \quad (22)$$

The above represents a diffraction pattern for photon 1 without any coincident counting, since integration over z_2 does not effect the z_1 dependent terms. It also says that photon 2 has a Gaussian distribution. As expected, if the two photons are not entangled, first order diffraction *will be visible* for photon 1, and photon 2 will not show any ghost diffraction. For $\beta \neq 0$, entanglement will kill any first order diffraction.

Another way to understand the missing first order interference is that by virtue of entanglement, each path of the photon 1, within the width of the slit, is correlated with a particular path of photon 2. By measuring the position of photon 2, one can, in principle, find out which particular path photon 1 had taken. If which-path information exists, no interference between different paths is possible, by virtue of Bohr's complementarity principle [17].

4 Conclusion

We have theoretically analysed the single-slit ghost diffraction experiment with entangled photons by using Fourier wave-packet evolution. At the heart of analysis is a generalised EPR state which describes the entangled photons. The effect of the slit is assumed to be a truncation of the wavefunction of the photon to within the width of the slit. Subsequent evolution is free. This simple analysis quantitatively reproduces all features of single-slit ghost diffraction. It explains why the diffraction pattern shifts if the detector D1 is fixed at a location other than $z_1 = 0$. It explains the strange distance that appears in the fringe-width formula of the ghost diffraction, even though the photon never actually travels that distance. This analysis also explains why first order diffraction behind the single slit is not observed.

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